Rules for Fractions

You may always multiply or divide **both** numerator and denominator by the same number … the new fraction is equivalent to the old one.
Examples:

$\frac{3}{7} = \frac{3\times5}{7\times5} = \frac{15}{35}$ $\frac{6}{9} = \frac{6\div3}{9\div3} = \frac{2}{3}$

To write an **improper fraction as a mixed numeral** divide the numerator by denominator … the denominator stays the same, write the whole part then the remainder over the denominator.
Example:

$write\ \frac{11}{4}\ as\ a\ mixed\ numeral$ ...

$11 \div 4 = 2\ remainder\ 3, \quad so \quad \frac{11}{4} = 2\frac{3}{4}$

To write a **mixed numeral as an improper fraction** multiply the whole part by the denominator, and add to the numerator
Example:

$2\frac{3}{5} = \frac{3+(2\times5)}{5} = \frac{13}{5}$

To **simplify a fraction** keep dividing numerator and denominator by the same number, until they are as small as possible (you should divide by the Highest Common Factor of the two numbers if you can find it first) … then, if it is an improper fraction write it as a mixed numeral.
Examples:

$\frac{450}{100} = \frac{450\div10}{100\div10} = \frac{45\div5}{10\div5} = \frac{9}{2} = 4\frac{1}{2}$ $since\ 9 \div 2 = 4\ remainder\ 1$

$\frac{12}{15} = \frac{12\div3}{15\div3} = \frac{4}{5}$

To find the **reciprocal** of a fraction, turn it upside down. Write a mixed numeral as an improper fraction first, then turn it upside down. After turning a proper fraction upside down it will be an improper fraction, simplify it by writing it as a mixed numeral.
Examples:

The reciprocal of $\frac{3}{5} = \frac{5}{3} = 1\frac{2}{3}$ $since\ 5 \div 3 = 1\ remainder\ 2$

The reciprocal of $1\frac{2}{5}$ first: $1\frac{2}{5} = \frac{2+(1\times5)}{5} = \frac{7}{5}$ then: $\text{the reciprocal of } \frac{7}{5} = \frac{5}{7}$

Fraction operations

To **add or subtract** two fractions, first find the "common denominator" …
write each one with the same denominator, then add or subtract the numerators.
Examples:

$$\frac{2}{5}+\frac{3}{7}=\frac{2\times7}{5\times7}+\frac{3\times5}{7\times5}=\frac{14+15}{5\times7}=\frac{29}{35}$$

$$1\frac{3}{4}-\frac{2}{3}=\frac{3+(1\times4)}{4}-\frac{2}{3}=\frac{7}{4}-\frac{2}{3}=\frac{7\times3}{4\times3}-\frac{2\times4}{3\times4}=\frac{21-8}{4\times3}=\frac{13}{12}=1\frac{1}{12}$$

Notice how I found a common denominator by multiply the denominators together.

If you have mixed numerals you may either convert to improper fractions first, or do the whole parts and fraction parts separately … but be very careful with subtraction!

To **multiply two fractions** multiply the numerators and denominators separately, write any mixed numerals as improper fractions before multiplying. Finally simplify the answer as far as possible.
Examples:

$$\frac{2}{5}\times\frac{3}{7}=\frac{2\times3}{5\times7}=\frac{6}{35}$$

$$1\frac{2}{3}\times\frac{3}{4}=\frac{3+2}{3}\times\frac{3}{4}=\frac{5}{3}\times\frac{3}{4}=\frac{15}{12}=\frac{15\div3}{12\div3}=\frac{5}{4}=1\frac{1}{4}$$

To **divide a fraction by another** multiply by the reciprocal
(so … turn the second upside-down and change the divide to a multiply)
Examples:

$$\frac{1}{2}\div\frac{1}{4}=\frac{1}{2}\times\frac{4}{1}=\frac{1\times4}{2\times1}=\frac{4}{2}=\frac{4\div2}{2\div2}=\frac{2}{1}=2$$

$$\frac{2}{5}\div\frac{2}{3}=\frac{2}{5}\times\frac{3}{2}=\frac{2\times3}{5\times2}=\frac{6}{10}=\frac{6\div2}{10\div2}=\frac{3}{5}$$

$$1\frac{1}{3}\div4=\frac{1+(1\times3)}{3}\div\frac{4}{1}=\frac{4}{3}\times\frac{1}{4}=\frac{4\times1}{3\times4}=\frac{4}{12}=\frac{4\div4}{12\div4}=\frac{1}{3}$$

Rules for Percentages

To write a **percentage as a fraction** … write it over 100, as a fraction, then simplify.
Examples:

$$325\% = \frac{325}{100} = \frac{325\div25}{100\div25} = \frac{13}{4} = 3\frac{1}{4}$$ (see the improper to mixed numerals rule)

$$70\% = \frac{70}{100} = \frac{70\div10}{100\div10} = \frac{7}{10}$$

To write **any number as a percentage** … multiply it by 100% (write the % sign!)
Examples:

$$0.2 = 0.2 \times 100\% = 20\%$$

$$\frac{3}{5} = \frac{3}{5} \times 100\% = \frac{3}{5} \times \frac{100}{1}\% = \frac{300}{5}\% = \frac{300\div5}{5\div5}\% = \frac{60}{1}\% = 60\%$$

To find a "**fraction of**" something, multiply by the fraction, possibly changing units.
Example:

$$\frac{3}{5}\ of\ 1\ hour = \frac{3}{5} \times 60\ minutes = \frac{3}{5} \times \frac{60}{1} minutes$$
$$= \frac{3\times60}{5} minutes = \frac{3\times60\div5}{5\div5} minutes = 36\ minutes$$

To find the "**percentage of**" something make the percentage a fraction then multiply
Examples:

75% of one dozen eggs $= \frac{75}{100} \times \frac{12}{1} = \frac{75\times12}{100} = \frac{900}{100} = \frac{900\div100}{100\div100} = \frac{9}{1} = 9$

More fraction facts

You can always write "1" as the denominator of a whole number so: $5 = \frac{5}{1}$
Then you can use the methods for fractions when given whole numbers in the question.

It is easier to compare two fractions if they are over the same denominator so …

$$Is\ \ \frac{5}{8} > \frac{11}{16}\ ?\quad working:\quad \frac{5}{8} = \frac{5\times2}{8\times2} = \frac{10}{16} < \frac{11}{16}\quad and\ the\ answer\ is\ \ldots\ NO$$