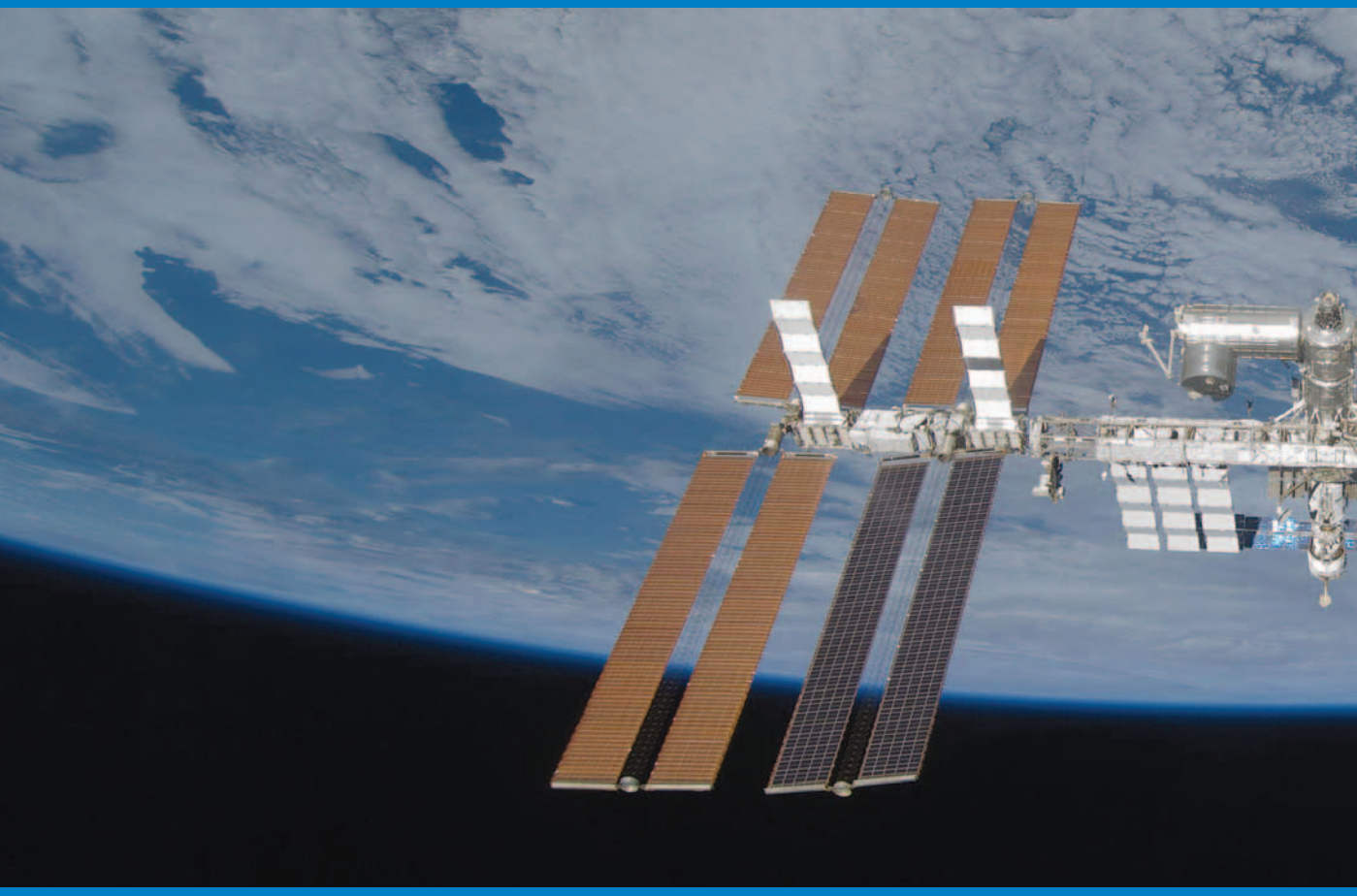
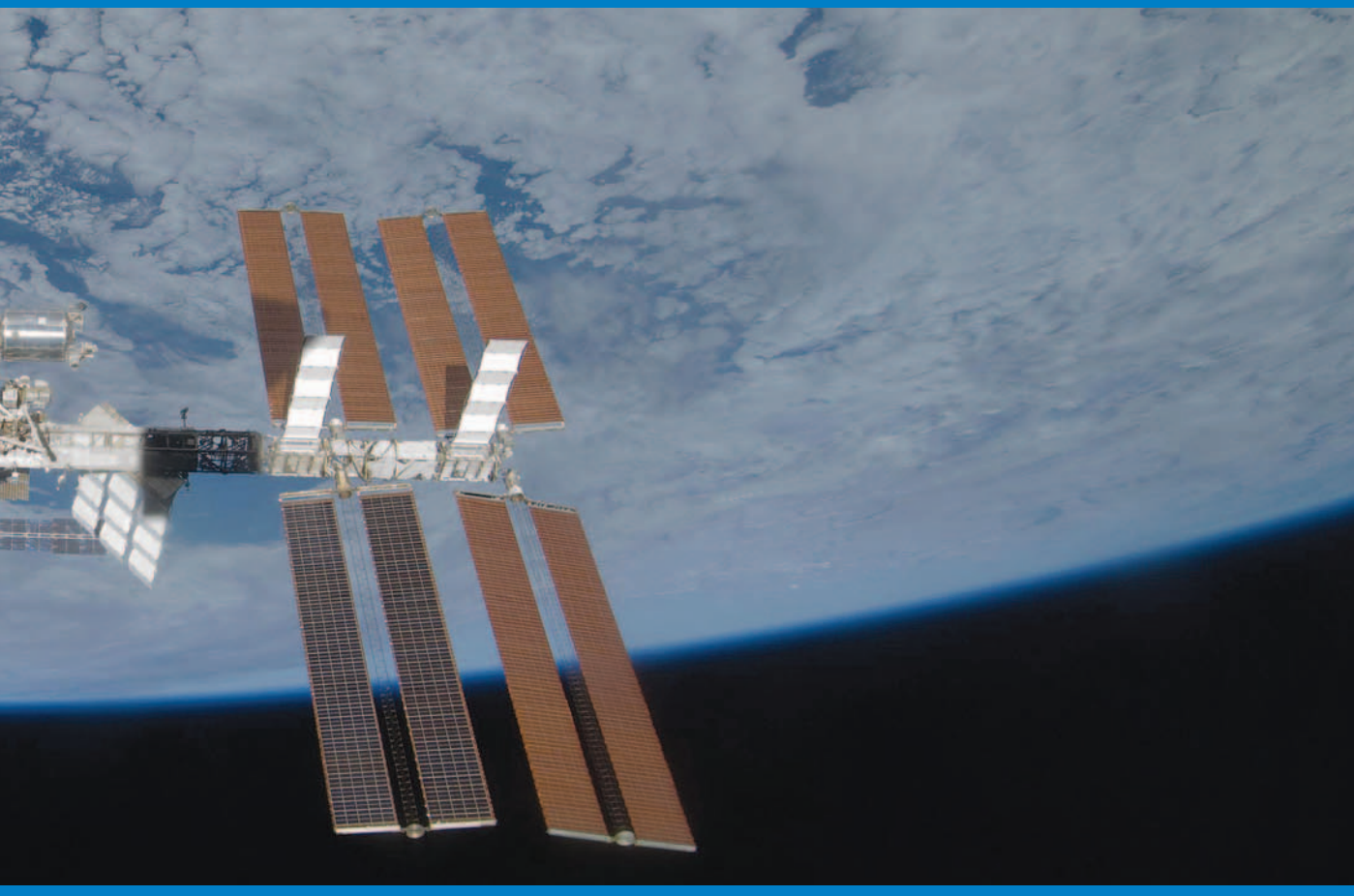


5

Number and algebra

Algebra and equations

Algebra is the branch of mathematics that uses letters of the alphabet to write general rules called **formulas**. Algebra is a powerful tool that has been used by mathematicians for over 4000 years. It was first used in ancient Babylon and Egypt in 2000 BCE and was brought to Europe from India by the Arabs during the 9th century. The word algebra comes from the Arabic *al-jabr*, meaning restoration. Algebra provides a method of solving equations and today it has a wide range of uses: from putting a satellite into space to predicting the chances that it will rain this weekend.



Chapter outline

	Proficiency strands			
5-01 The laws of arithmetic	U	F		R
5-02 The distributive law	U	F		R
5-03 Variables	U	F		R
5-04 From words to algebraic expressions	U	F	PS	R
5-05 Substitution	U	F	PS	
5-06 Equations	U	F		R
5-07 One-step equations	U	F		R
5-08 Two-step equations	U	F		R
5-09 Equation problems		F	PS	R

Wordbank

commutative law A law of arithmetic that says that numbers can be added or multiplied in reverse order, for example, $8 + 3 = 3 + 8$, $5 \times 9 = 9 \times 5$

formula A general mathematical rule written using letters and symbols

inverse operation An opposite operation used in solving an equation, for example, the inverse operation of multiplying is dividing

solve (an equation) To find the value of an unknown variable in an equation

substitute To replace a variable with a number

variable A quantity that can take on different values, represented by a symbol such as a letter of the alphabet

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In this chapter you will:

Weblink
Algebra masterclass

- apply the associative, commutative and distributive laws to aid mental and written computation
- understand that arithmetic laws provide powerful ways of describing and simplifying calculations and that using these laws leads to the generality of algebra
- introduce the concept of variables as a way of representing numbers using letters
- extend and apply the laws and properties of arithmetic to algebraic terms and expressions
- move fluently between algebraic and word representations as descriptions of the same situation
- create algebraic expressions and evaluate them by substituting a given value for each variable
- use authentic formulas to perform substitutions
- solve simple linear equations
- solve real-life problems by using pronumerals to represent unknowns

SkillCheck

Worksheet
StartUp assignment 5
MAT07NAWK10035

1 If ● = 3, then find the value of each expression.

a ● + 5 **b** 7 − ● **c** 2 × ● **d** 12 + ●
e ● − 1 **f** ● × 10 **g** ● + ● **h** 27 ÷ ●
i −6 + ● **j** ● × (−1) **k** −3 ÷ ● **l** −4 − ●

2 Find the value of ■ in each equation.

a ■ + 4 = 10 **b** 6 × ■ = 36 **c** ■ + ■ = 20
d 18 − ■ = 11 **e** 12 ÷ ■ = −3 **f** ■ − 6 = 1

3 Write the number that is:

a 4 more than 7 **b** double 10
c the next odd number after 9 **d** half of 18
e 10 less than 14 **f** the difference between 8 and 1
g 5 multiplied by itself **h** 5 divided by itself
i the amount by which 11 is greater than 9 **j** the product of 4 and 8

4 Evaluate each product without using a calculator.

a 15 × 10 **b** 28 × 9 **c** 7 × 4 × 5
d 16 × 5 **e** 13 × 10 **f** 13 × 2

5 Is 13 × 12 the same as 13 × 10 + 13 × 2?

6 **a** If ▲ = −7, evaluate 2 × ▲ + 3 **b** If ● = 4, evaluate 3 × ● + 5
c If ● = 15, evaluate ● ÷ 3 − 4 **d** If ▲ = 5, evaluate 4 × ▲ − 6
e If ■ = −2, evaluate 9 × 2 + ■ **f** If ■ = 6, evaluate 4 × 3 − ■
g If ● = −4, evaluate ● × 4 + 3 **h** If ▲ = −1, evaluate 10 × 2 − ▲

7 Evaluate each expression.

a 5 − 8 **b** −8 + 3 **c** 3 × (−4) **d** −6 ÷ 3
e −2 + 5 **f** −2 − 9 **g** −4 × (−9) **h** −12 ÷ (−4)
i −3 − 7 **j** 80 ÷ (−10) **k** 6 − 15 **l** −8 × 9

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5-01 The laws of arithmetic

Arithmetic is the branch of mathematics involved in calculating with **numbers**, involving the four operations of adding, subtracting, multiplying and dividing. We will now look at some general rules of arithmetic and use algebraic symbols to write these rules formally.

The commutative laws

The **commutative law** says that you can operate with two numbers in reverse order. For example:

$4 + 7 = 11$ and $7 + 4 = 11$, so $4 + 7 = 7 + 4$.
$9 \times 5 = 45$ and $5 \times 9 = 45$, so $9 \times 5 = 5 \times 9$.

The commutative law works for **adding** or **multiplying** only. It does not work for subtracting or dividing. For example, $8 - 2$ is not equal to $2 - 8$ and $14 \div 2$ is not equal to $2 \div 14$.

Summary

If a and b stand for numbers, then:

$$a + b = b + a$$
$$a \times b = b \times a$$

The associative laws

The **associative law** says that when operating with more than 2 numbers, you can group them in any way. For example:

$$\begin{aligned} 6 + 7 + 4 &= (6 + 7) + 4 \\ &= 13 + 4 \\ &= 17 \end{aligned}$$ and $$\begin{aligned} 6 + 7 + 4 &= 6 + (7 + 4) \\ &= 6 + 11 \\ &= 17 \end{aligned}$$

so $(6 + 7) + 4 = 6 + (7 + 4)$.

$$\begin{aligned} 5 \times 3 \times 2 \times 2 &= (5 \times 3) \times 2 \times 2 \\ &= 15 \times 2 \times 2 \\ &= 30 \times 2 \\ &= 60 \end{aligned}$$ and $$\begin{aligned} 5 \times 3 \times 2 \times 2 &= 5 \times (3 \times 2) \times 2 \\ &= 5 \times 6 \times 2 \\ &= 30 \times 2 \\ &= 60 \end{aligned}$$

so $(5 \times 3) \times 2 \times 2 = 5 \times (3 \times 2) \times 2$.

The associative law only works for **adding** or **multiplying**, not for subtracting or dividing. For example, $10 - 3 - 1$ is not equal to $10 - (3 - 1)$ and $20 \div 5 \div 2$ is not equal to $20 \div (5 \div 2)$.

Summary

If a, b and c stand for numbers, then:

$$(a + b) + c = a + (b + c)$$
$$(a \times b) \times c = a \times (b \times c)$$

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The commutative and associative laws of arithmetic can be used to help us with mental calculations.

Example 1

Evaluate each sum.

a $6 + 16$ **b** $26 + 15 + 4$ **c** $58 + 55 + 45$ **d** $-8 + 22$

Solution

a $6 + 16 = 16 + 6$
$= 22$

It is easier to add smaller numbers to larger ones, so swap 6 and 16 around.

b When adding, pair numbers together that add to multiples of 10, 100, and so on.

$26 + 15 + 4 = (26 + 4) + 15$
$= 30 + 15$
$= 45$

26 and 4 add to 30, so put them together.

c $58 + 55 + 45 = 58 + (55 + 45)$
$= 58 + 100$
$= 158$

55 and 45 add to 100, so put them together.

d $-8 + 22 = 22 + (-8)$
$= 22 - 8$
$= 14$

It is easier to work with larger and positive numbers first, so swap –8 and 22 around.

Example 2

Evaluate each product.

a $17 \times 5 \times 2$ **b** $25 \times 9 \times 4$

Solution

When multiplying, pair together numbers that multiply to 10, 100, 1000 and so on.

a $17 \times 5 \times 2 = 17 \times (5 \times 2)$
$= 17 \times 10$
$= 170$

$5 \times 2 = 10$, so put 5 and 2 together.

b $25 \times 9 \times 4 = (25 \times 4) \times 9$
$= 100 \times 9$
$= 900$

$25 \times 4 = 100$, so put 25 and 4 together.

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Multiplying by a multiple of 10

Example 3

Evaluate each product.

a 9×50 **b** 6×400 **c** 20×300

Solution

You can factorise multiples of 10, 100, 1000 and so on to make multiplying easier.

a $9 \times 50 = 9 \times 5 \times 10$
$= 45 \times 10$
$= 450$

b $6 \times 400 = 6 \times 4 \times 100$
$= 24 \times 100$
$= 2400$

c $20 \times 300 = 2 \times 10 \times 3 \times 100$
$= 2 \times 3 \times 10 \times 100$
$= 6 \times 1000$
$= 6000$

Exercise 5-01 The laws of arithmetic

1 Evaluate each sum. *See Example 1*

a $4 + 21$ **b** $19 + 99$ **c** $18 + 23 + 7$
d $75 + 50 + 25$ **e** $16 + 80 + 44 + 10$ **f** $56 + 5 + 15 + 4$
g $-18 + 28$ **h** $16 + 45 + 4$ **i** $-6 + 20$
j $81 + 7 + 19 + 12$ **k** $54 + 27 + 16 + 3$ **l** $44 + 12 + 16 + 12$

2 Evaluate each product. *See Example 2*

a $2 \times 17 \times 5$ **b** $81 \times 2 \times 5$ **c** $50 \times 14 \times 2$
d $10 \times 32 \times 2$ **e** $5 \times 7 \times 4$ **f** $2 \times 5 \times 3 \times 5$
g $8 \times 3 \times 2$ **h** $25 \times 8 \times 2$ **i** $4 \times 4 \times 5$
j $20 \times 6 \times 5$ **k** $3 \times 2 \times 5 \times 3$ **l** $2 \times 14 \times 5 \times 10$

3 Evaluate each product. *See Example 3*

a 7×1000 **b** 100×100 **c** 3×90 **d** 4×700
e 5×80 **f** 2×600 **g** 4×40 **h** 12×2000
i 4×6000 **j** 3×1100 **k** 90×20 **l** 5×500
m 8×40 **n** 20×20 **o** 300×70 **p** 700×300

4 Evaluate $18 \times 4 \times 5$. Select the correct answer **A**, **B**, **C** or **D**.

A 110 **B** 162 **C** 180 **D** 360

5 Evaluate 60×600. Select the correct answer **A**, **B**, **C** or **D**.

A 1200 **B** 3600 **C** 12 000 **D** 36 000

Worked solutions
Exercise 5-01
MAT07NAWS10028

Worked solutions
Exercise 5-01
MAT07NAWS10028

6 Write True (T) or False (F) for each equation.

a $4 - 8 = 8 - 4$ **b** $6 \times 5 = 5 \times 6$ **c** $10 + 11 = 11 + 10$
d $12 \div 4 = 4 \div 12$ **e** $a + 5 = 5 + a$ **f** $3 - x = x - 3$
g $b \div 2 = 2 \div b$ **h** $c \times 2 = 2 \times c$ **i** $x - y = y - x$
j $a \times b = b \times a$ **k** $x + y = y + x$ **l** $c \div d = d \div c$

5-02 The distributive law

Homework sheet
Mental calculation
MAT07NAHS10018

The **distributive law** says that you can multiply by a number by splitting it into the sum or difference of two other numbers. Look at these two examples.

a $17 \times 12 = 17 \times (10 + 2)$
$= (10 + 2) + (10 + 2) + (10 + 2) + \cdots$ 17 times
$= 10 + 10 + 10 + \cdots + 2 + 2 + 2 + \cdots$ 17 times each
$= (17 \times 10) + (17 \times 2)$
$= 170 + 34$
$= 204$
so $17 \times (10 + 2) = 17 \times 10 + 17 \times 2$

b $14 \times 9 = 14 \times (10 - 1)$
$= (10 - 1) + (10 - 1) + (10 - 1) + \cdots$ 14 times
$= 10 + 10 + 10 + \cdots - 1 - 1 - 1 - \cdots$ 14 times each
$= (14 \times 10) - (14 \times 1)$
$= 140 - 14$
$= 126$
so $14 \times (10 - 1) = 14 \times 10 - 14 \times 1$

Summary

If a, b and c stand for numbers, then:

$$a \times (b + c) = a \times b + a \times c$$
$$a \times (b - c) = a \times b - a \times c$$

The distributive law of arithmetic sometimes helps us multiply numbers more easily.

Example 4

Video tutorial
Multiplying by 8, 9, 11 and 12
MAT07NAVT10010

Evaluate each product.

a 25×9 **b** 18×8

Solution

a $25 \times 9 = 25 \times (10 - 1)$
$= 25 \times 10 - 25 \times 1$
$= 250 - 25$
$= 225$

b $18 \times 8 = 18 \times (10 - 2)$
$= 18 \times 10 - 18 \times 2$
$= 180 - 36$
$= 144$

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Summary

- **To multiply a number by 9**, multiply by 10 and then subtract the number
- **To multiply a number by 8**, multiply by 10 and then subtract double the number

Example 5

Evaluate each product.

a 13×11 **b** 27×12

Video tutorial
Multiplying by 8, 9, 11 and 12
MAT07NAVT10010

Solution

a $13 \times 11 = 13 \times (10 + 1)$
$= 13 \times 10 + 13 \times 1$
$= 130 + 13$
$= 143$

b $27 \times 12 = 27 \times (10 + 2)$
$= 27 \times 10 + 27 \times 2$
$= 270 + 54$
$= 324$

Summary

- **To multiply a number by 11**, multiply by 10 and then add the number
- **To multiply a number by 12**, multiply by 10 and then add double the number

Exercise 5-02 The distributive law

1 Copy and complete each multiplication.

a $18 \times 12 = 18 \times (10 + \_\_\_\_)$
$= 18 \times \_\_\_\_ + 18 \times \_\_\_\_$
$= \_\_\_\_ + 36$
$= \_\_\_\_$

b $16 \times 9 = \_\_\_\_ \times (\_\_\_\_ - 1)$
$= \_\_\_\_ \times 10 - \_\_\_\_ \times 1$
$= 160 - \_\_\_\_$
$= \_\_\_\_$

c $21 \times 11 = 21 \times (\_\_\_\_ + \_\_\_\_)$
$= \_\_\_\_ \times 10 + 21 \times \_\_\_\_$
$= 210 + \_\_\_\_$
$= \_\_\_\_$

d $15 \times 8 = 15 \times (10 - \_\_\_\_)$
$= \_\_\_\_ \times 10 - \_\_\_\_ \times \_\_\_\_$
$= \_\_\_\_ - \_\_\_\_$
$= \_\_\_\_$

2 Evaluate each product. See Example 4

a 15×9 **b** 18×9 **c** 26×9 **d** 31×9
e 16×8 **f** 13×8 **g** 22×8 **h** 14×8
i 27×9 **j** 45×8 **k** 38×8 **l** 29×9

See Example 5

3 Evaluate each product.

a 17×11	**b** 24×11	**c** 15×11	**d** 29×11
e 25×12	**f** 14×12	**g** 24×12	**h** 33×12
i 40×12	**j** 12×11	**k** 9×12	**l** 19×11

4 Use the distributive law to copy and complete each multiplication.

a $6 \times 22 = 6 \times (20 + \_\_\_\_)$
$= 6 \times \_\_\_\_ + 6 \times \_\_\_\_$
$= 120 + \_\_\_\_$
$= \_\_\_\_$

b $12 \times 19 = \_\_\_\_ \times (20 - \_\_\_\_)$
$= \_\_\_\_ \times 20 - \_\_\_\_ \times 1$
$= 240 - \_\_\_\_$
$= \_\_\_\_$

c $40 \times 41 = \_\_\_\_ \times (\_\_\_\_ + 1)$
$= \_\_\_\_ \times 40 + \_\_\_\_ \times \_\_\_\_$
$= 1600 + \_\_\_\_$
$= \_\_\_\_$

d $25 \times 48 = \_\_\_\_ \times (50 - \_\_\_\_)$
$= \_\_\_\_ \times 50 - \_\_\_\_ \times \_\_\_\_$
$= \_\_\_\_ - \_\_\_\_$
$= \_\_\_\_$

Worked solutions
Exercise 5-02
MAT07NAWS10029

5 Use the distributive law to evaluate each product.

a 5×32	**b** 4×51	**c** 12×99	**d** 8×102
e 6×29	**f** 12×21	**g** 15×18	**h** 11×49

6 Which is the correct distributive law for calculating 14×12? Select the correct answer **A**, **B**, **C** or **D**.

A $14 \times 10 \times 2$
B $14 \times 10 + 2$
C $10 + 4 \times 10 + 2$
D $14 \times 10 + 14 \times 2$

5-03 Variables

Algebra uses letters or symbols to write general mathematical rules called **formulas** or **equations**. We have used algebra to write the laws of arithmetic we have learnt.

The commutative laws:

- $a + b = b + a$
- $a \times b = b \times a$

The associative laws:

- $(a + b) + c = a + (b + c)$
- $(a \times b) \times c = a \times (b \times c)$

The distributive law:

- $a \times (b + c) = a \times b + a \times c$
- $a \times (b - c) = a \times b - a \times c$

In the formulas above, a, b and c represent any three numbers and are called **variables** or **pronumerals**. In algebra, a variable or pronumeral is a symbol, usually a letter of the alphabet, that represents a number. It is called a **variable** because its value can vary (change), and a **pronumeral** because it stands in place of a numeral.

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Example 6

Examine each of the following number patterns, then write the general rule using a variable.

a
$4 \times 1 = 4$
$7 \times 1 = 7$
$9 \times 1 = 9$
$1 \times 1 = 1$
$6 \times 1 = 6$
$b \times 1 =$ ____

b
$9 + 9 + 9 = 3 \times 9$
$4 + 4 + 4 = 3 \times 4$
$11 + 11 + 11 = 3 \times 11$
$2 + 2 + 2 = 3 \times 2$
$5 + 5 + 5 = 3 \times 5$
$x + x + x =$ ____

c
$6 - 6 = 0$
$1 - 1 = 0$
$17 - 17 = 0$
$12 - 12 = 0$
$4 - 4 = 0$

d
$2 \times (-2) = -(2^2)$
$8 \times (-8) = -(8^2)$
$5 \times (-5) = -(5^2)$
$9 \times (-9) = -(9^2)$
$1 \times (-1) = -(1^2)$

Solution

The general rule is:

a $b \times 1 = b$ — Any number multiplied by 1 is itself.

b $x + x + x = 3 \times x$ — The sum of the same number 3 times is 3 times that number.

c $a - a = 0$ — Other variables are possible, for example, $k - k = 0$ — Any number minus itself is 0.

d $r \times (-r) = -(r^2)$ — Other variables are possible, for example, $y \times (-y) = -(y^2)$ — Any number multiplied by its opposite is equal to the negative of that number squared.

Algebraic abbreviations

Mathematicians prefer to write expressions as simply as they can. For example:
$5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 = 9 \times 5$ and $2 \times 2 \times 2 \times 2 \times 2 = 2^5$.
When writing algebraic expressions, we use the following abbreviations:

- $3 \times k = 3k$ — for multiplication, we leave out the '$\times$' symbol
- $m \div 4 = \frac{m}{4}$ — for division, we can write in fraction form
- $r \times r = r^2$ — for powers
- $1x = x$ — for multiplying by 1
- $c \times a \times 6 = 6ac$ — write the number first, then the variables in alphabetical order

Video tutorial
Algebraic expressions
MAT07NAVT10011

Puzzle sheet
Algebra bingo game
MAT07NAPS10017

Example 7

Simplify each expression.

a $d \times 8$ **b** $m + m + m + m + m$ **c** $k \div 2$
d $u \times 9 \times w$ **e** $6x + 2x$ **f** $3 \times p \times 2 \times h$
g $z \times z \times z$ **h** $3n - 2n$ **i** $y \times y \times 4$

Solution

a $d \times 8 = 8d$ Write the number first.

b $m + m + m + m + m = 5m$

c $k \div 2 = \frac{k}{2}$

d $u \times 9 \times w = 9uw$ Write the number first, then the variables in alphabetical order.

e $6x + 2x = 8x$ $(x + x + x + x + x + x) + (x + x) = 8$ lots of x

f $3 \times p \times 2 \times h = 3 \times 2 \times p \times h$
$= 6hp$

g $z \times z \times z = z^3$

h $3n - 2n = 1n$ $(n + n + n) - (n + n) =$ one lot of n
$= n$ There is no need to write the '1'.

i $y \times y \times 4 = 4y^2$ The '2' (squared) belongs to the y only.

Skillsheet
Order of operations
MAT07NASS10005

Example 8

Use order of operations to simplify each expression.

a $2 \times r + 4$ **b** $15 - d \div 3$ **c** $7 \div (n \times n) + 1$

Solution

a $2 \times r + 4 = 2r + 4$ $\times$ first, then $+$

b $15 - d \div 3 = 15 - (d \div 3)$ $\div$ first, then $-$
$= 15 - \frac{d}{3}$

c $7 \div (n \times n) + 1 = 7 \div n^2 + 1$ ( ) first, then $\div$, then $+$
$= \frac{7}{n^2} + 1$

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Example 9

Write each expression in expanded form.

a $3ab$ **b** $-8st^2$ **c** $x^2 - \frac{y}{10}$

Solution

a $3ab = 3 \times a \times b$ **b** $-8st^2 = -8 \times s \times t \times t$ **c** $x^2 - \frac{y}{10} = x \times x - y \div 10$

Note that expanding is the opposite of simplifying.

Exercise 5-03 Variables

1 Examine each number pattern, then write the general rule using a variable. A clue has been given in parts **a** and **b**. *See Example 6*

a
$8 \div 8 = 1$
$1 \div 1 = 1$
$-2 \div (-2) = 1$
$14 \div 14 = 1$
$-7 \div (-7) = 1$
$u \div u =$ ____

b
$6 + 6 = 2 \times 6$
$10 + 10 = 2 \times 10$
$4 + 4 = 2 \times 4$
$9 + 9 = 2 \times 9$
$-4 + (-4) = 2 \times (-4)$
$p + p =$ ____

c
$3 + (-3) = 0$
$7 + (-7) = 0$
$20 + (-20) = 0$
$1 + (-1) = 0$
$16 + (-16) = 0$

d
$2 \times 0 = 0$
$19 \times 0 = 0$
$-3 \times 0 = 0$
$0 \times 0 = 0$
$-24 \times 0 = 0$

e
$7 + 0 = 7$
$-7 + 0 = -7$
$3 + 0 = 3$
$11 + 0 = 11$
$-100 + 0 = -100$

f
$-5 \times (-5) = 5^2$
$-7 \times (-7) = 7^2$
$-12 \times (-12) = 12^2$
$-1 \times (-1) = 1^2$
$-8 \times (-8) = 8^2$

2 Simplify each expression. *See Example 7*

a $w \times 3$ **b** $b \times b$ **c** $9 \times c \times a \times d$
d $a + a + a + a + a + a$ **e** $e \div 5$ **f** $f + f$
g $5m + 2m$ **h** $2 \times w \times 3 \times h$ **i** $5m - 2m$
j $4 \times f \times f$ **k** $6d - 5d$ **l** $a \times c \times b \times 10$
m $16 \div n$ **n** $g \times 2 \times g$ **o** $3x + 6x$
p $h + h - h$ **q** $12q - 4q$ **r** $2 \times m \times 4 \times n$
s $6 \times p \times r \times (-2) \times r$ **t** $a + a + a + b + b$ **u** $a \times a \times a \times b \times b$

Worked solutions
Exercise 5-03
MAT07NAWS10030

3 Use order of operations to simplify each expression. *See Example 8*

a $16 \div (a \times 5)$ **b** $7 + 3 \times n$ **c** $(e - 6) \div 2$
d $4 \times s - 8$ **e** $c \times c + d \times d$ **f** $k \div 9 + m$
g $k \div (9 + m)$ **h** $12 - r \div 2$ **i** $7 + u \times u \times 7$

4 Which of the following is equal to $m \times m \times 3 + 4$? Select the correct answer **A**, **B**, **C** or **D**.

A $3m^2 + 4$ **B** $2m^3 + 4$ **C** $6m + 4$ **D** $m^2 + 7$

5 Explain the meaning of:

a xy **b** $\frac{2n}{r}$ **c** $1x = x$

6 Which of the following is not equal to $2p$? Select the correct answer **A**, **B**, **C** or **D**.

A $3p - p$ **B** p^2 **C** $p + p$ **D** $p \times 2$

See Example 9

7 Write each expression in expanded form.

a $12rs$ **b** $-2n^2$ **c** $\frac{8}{f}$ **d** $r^2 + t^2$

e $9a^2b$ **f** $5mn - 2a$ **g** $\frac{x+1}{3}$ **h** q^3

i $14 - 2d^2$ **j** $\frac{-4c}{5}$ **k** x^2y^2 **l** $4j - \frac{8}{k}$

8 What is $4m^3$ in expanded form? Select the correct answer **A**, **B**, **C** or **D**.

A $4m \times 4m \times 4m$ **B** $4m + 4m + 4m$ **C** $4 \times m + m + m$ **D** $4 \times m \times m \times m$

Investigation: The laws of arithmetic

We can use algebraic symbols to describe general laws about arithmetic.
For example, if we add zero to any number, the answer is still that number.
If we let N stand for any number: $N + 0 = N$
In groups of two to four students, answer the following questions.

1 Describe in words what each number rule below means.

a $N \times 1 = N$ **b** $N \times 0 = 0$

c $a - b \neq b - a$ **d** $a + b + c = b + a + c$

e $N - 0 = N$ **f** $ab = ba$

2 Write each of the following rules algebraically using variables.

a Any number divided by 1 equals itself.

b Multiplying a number by 8 is the same as doubling it three times.

c Any three numbers can be multiplied together in any order.

d Any number added to itself is the same as multiplying that number by 2.

e Any number subtracted from itself equals 0.

f Any number multiplied by its reciprocal equals 1.

3 Is each of the following equations true or false? Test your decision by substituting a number for the variable and checking.

a $a \div b = b \div a$ **b** $N \div N = 1$

c $4a - a = 4$ **d** a is a factor of a

e If N is even, then $N + 3$ is odd **f** $\frac{1}{2}N = N - 2$

g $a + (-a) = 2a$ **h** $0 \div N = 0$

i $N \div 0 = 0$ **j** $N \div 1 = N$

4 If k is an odd number, what is an expression for:

a the previous odd number? **b** the next even number?

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5-04 From words to algebraic expressions

We will continue to use variables to convert worded expressions into algebraic expressions.

Puzzle sheet
What's the expression?
MAT07NAPS10018

Worksheet
Writing algebraic expressions
MAT07NAWK00002

Example 10

If A represents any number, write an algebraic expression for:

a three times that number
b three less than that number
c the next consecutive number
d that number multiplied by itself
e the square root of that number
f one-third of that number

Consecutive means 'following in order': for example, 7, 8, 9 are consecutive numbers

Solution

a $3 \times A = 3A$
b $A - 3$
c $A + 1$

The next number after A is $A + 1$
(for example, the next number after 4 is $4 + 1 = 5$)

d $A \times A = A^2$
e $\sqrt{A}$
f $\frac{1}{3}A = A \div 3 = \frac{A}{3}$

Example 11

Write an expression for:

a the sum of m and 5
b double k
c u increased by 10
d the difference between p and q
e the product of r and 12
f how many times y divides into 50

Solution

a $m + 5$
b $2k$
c $u + 10$
d $p - q$
e $12r$
f $50 \div y = \frac{50}{y}$

Exercise 5-04 From words to algebraic expressions

See Example 10

1 If N represents any number, write an algebraic expression for:

a the number added to itself
b half the number
c triple the number
d 7 less than the number
e one-tenth of the number
f the previous consecutive number
g 5 times the number
h the sum of the number and 21
i the difference between the number and 1
j the number divided by 8
k the number increased by 3
l the number cubed
m the square root of the number

2 Suppose that in question **1**, the variable X represents any number instead of N. What difference would this make to your answers? Does it matter which letter of the alphabet you choose to use?

See Example 11

3 Write an expression for:

a the sum of all x, y and z
b the difference between b and c
c the product of u and v
d the product of u, v and v
e the quotient of C and d
f the sum of m and n, divided by 2
g the square root of the product of 10 and a
h 3 less than b
i 5 decreased by c
j c decreased by 5
k the number of times that 3 divides into e
l t squared
m the product of x, z and 9
n how much k is more than 12
o half of the difference between 20 and g

4 Write an expression for:

a the number of students in a class if there are b boys and g girls
b the number of chicken nuggets needed for a party if there are n guests and each guest eats an average of 4 nuggets
c the number of children left in a class of d students if r of them have gone to the zone swimming carnival
d the amount of money earned selling s sushi rolls at the canteen, where each roll is sold at \$2
e the cost of one movie ticket if it costs \$$a$ for three tickets
f the perimeter of a rectangle of length x and width y
g the area of a rectangle of length x and width y
h the total cost of buying m milkshakes and i ice creams, where each milkshake costs \$4 and each ice cream costs \$2

Worked solutions
Exercise 5-04
MAT07NAWS10031

5 What is the correct expression for the total admission cost to Water World for c children at \$21 each and a adults at \$27 each? Select the correct answer **A**, **B**, **C** or **D**.

A $21a + 27c$ **B** $48ac$ **C** $27a + 21c$ **D** $a + c + 48$

Worked solutions
Exercise 5-04
MAT07NAWS10031

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Mental skills 5A Maths without calculators

Multiplying an even number by 5

To multiply an even number by 5, it's easier to halve it, then multiply by 10 (by inserting a 0 at the end). This is because $\frac{1}{2} \times 10 = 5$.

1 Study each example.

a $14 \times 5 = 14 \times \frac{1}{2} \times 10 = 7 \times 10 = 70$

b $36 \times 5 = 36 \times \frac{1}{2} \times 10 = 18 \times 10 = 180$

c $22 \times 5 = 11 \times 10 = 110$

d $18 \times 5 = 9 \times 10 = 90$

2 Now simplify each product.

a 32×5	**b** 26×5	**c** 12×5	**d** 28×5
e 42×5	**f** 54×5	**g** 38×5	**h** 44×5
i 60×5	**j** 34×5	**k** 16×5	**l** 58×5

Technology Using variables and values

1 **a** Create the following spreadsheet to calculate the value of the expression $\boldsymbol{x + 2}$ for different values of x.

	A	B	C	D	E	F	G	H	I	J	K
1	x	1	2	3	4	5	10	20	25	50	100
2	x + 2										
3											

b To evaluate $x + 2$ when $x = 1$ (column B), enter $=$**B1**$+$**2** into cell B2. What value should you get?

	A	B	C	D	E	F	G	H	I	J	K
1	x	1	2	3	4	5	10	20	25	50	100
2	x + 2	=B1+2									
3											

c To complete the table, use **Fill Right** to copy the rule across row 2, from cell B2 until cell K2. **Centre** your answers in each of the cells. Check that the answers look correct.

d Type over some values in row A of the spreadsheet to find the value of $x + 2$ when:

i $x = 6$ **ii** $x = 17$ **iii** $x = 31$

e Predict what value of x will give the answer 96 and check your guess using the spreadsheet.

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2 **a** Create the following spreadsheet to calculate the value of $x - 3$ for values of x.

	A	B	C	D	E	F	G	H	I	J	K
1	x	1	2	3	4	5	10	20	25	50	100
2	x - 3										

b In cell B2, enter a formula beginning with =**B1** to calculate $x - 3$.

c Use **Fill Right** to copy this formula up to cell K2. **Centre** your answers.

d Type over some values in row A to evaluate $x - 3$ when:

i $x = 9$ **ii** $x = 28$

e Predict what value of x will give the answer:

i 37 **ii** 105

Check your guess using the spreadsheet.

3 **a** Create the following spreadsheet and in cell B2, enter a formula to evaluate $x \div 2$.

	A	B	C	D	E	F	G	H	I	J	K
1	x	-8	-6	-4	-2	0	2	10	20	30	40
2	x ÷ 2										

b Use **Fill Right** to copy this formula up to cell K2. **Centre** your answers.

c Use the spreadsheet to evaluate $x \div 2$ when:

i $x = -10$ **ii** $x = 6$

d Predict what value of x will give the answer 21 and check using the spreadsheet.

4 **a** Create the following spreadsheet and in cell B2, enter a formula to evaluate $10x - 1$.

	A	B	C	D	E	F	G
1	x	-10	-5	0	5	10	15
2	10x - 1						

b Use **Fill Right** to copy this formula up to cell G2. Centre your answers.

c Use the spreadsheet to evaluate $10x - 1$ when:

i $x = 8$ **ii** $x = -7$

d Predict what value of x will give the answer 199 and check using the spreadsheet.

5 **a** Create a new spreadsheet to evaluate x^2 for different values of x. In cell A1, enter the label 'x', then along row A enter the following values: -2, -1, 0, and so on until 4 is in cell H1.

b In cell A2, enter the label 'x^2' and in cell B2, enter a formula to evaluate x^2.

c **Fill Right** to complete the answers for cells C2 to H2.

Homework sheet
Algebra
MAT07NAHS10019

5-05 Substitution

Worksheet
Substitution
MAT07NAWK10036

The word **substitution** means replacing one thing with another thing. In sport, substitution means to replace one player with another during a game.
In algebra, substitution means to replace a variable with a number to evaluate an algebraic expression.

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Example 12

Puzzle sheet
Substitution game
MAT07NAPS10019

a Evaluate $4k - 9$ when $k = 15$
b If $r = m^2 + 3m$, evaluate r when $m = 4$
c If $a = 6$, $b = -4$ and $c = 10$, evaluate $\frac{c - a}{b}$

Solution

a When $k = 15$,

$$\begin{aligned} 4k - 9 &= 4 \times 15 - 9 \\ &= 51 \end{aligned}$$

b When $m = 4$,

$$\begin{aligned} r &= 4^2 + 3 \times 4 \\ &= 28 \end{aligned}$$

c When $a = 6$, $b = -4$ and $c = 10$,

$$\begin{aligned} \frac{c - a}{b} &= \frac{10 - 6}{-4} \\ &= \frac{4}{-4} \\ &= -1 \end{aligned}$$

Substituting into formulas

A **formula** is a general rule written as an algebraic equation showing the relationship between variables. Solving mathematical problems often involves substituting values into formulas.

Example 13

The formula for the perimeter of a rectangle is

$$P = 2l + 2w$$

where P is the perimeter, l is the length and w is the width.

Use this formula to calculate the perimeter of a rectangle with length 16 m and width 3 m.

Solution

When length $l = 16$ and width $w = 3$,

$$\begin{aligned} P &= 2l + 2w \\ &= 2 \times 16 + 2 \times 3 \\ &= 38 \text{ m} \end{aligned}$$

Exercise 5-05 Substitution

See Example 12

1 Evaluate $k + 3$ when:

a $k = 12$ **b** $k = 48$ **c** $k = 119$ **d** $k = -21$

2 Evaluate $45 - k$ when:

a $k = 5$ **b** $k = -13$ **c** $k = 28$ **d** $k = 45$

3 Evaluate $4k$ when:

a $k = 2$ **b** $k = 11$ **c** $k = -4$ **d** $k = \frac{1}{2}$

4 Find the value of $5k + 1$ when:

a $k = 3$ **b** $k = -4$ **c** $k = 20$ **d** $k = 7$

5 Find the value of $9k - 8$ if:

a $k = 9$ **b** $k = -5$ **c** $k = 0$ **d** $k = 2$

6 Find the value of $\frac{k}{3}$ if:

a $k = 15$ **b** $k = 33$ **c** $k = -9$ **d** $k = 57$

7 If $a = 6$, $b = 14$ and $c = -2$, find:

a $a + b - c$ **b** $2b + c$ **c** $\frac{a - b}{c}$ **d** $ab - 50$

8 If $m = 4$, $n = -2$ and $p = 3$, find:

a m^2 **b** $3n + p$ **c** mnp

Worked solutions
Exercise 5-05
MAT07NAWS10032

9 If $d = 4c - 10$, find d when:

a $c = 5$ **b** $c = 8$ **c** $c = -10$

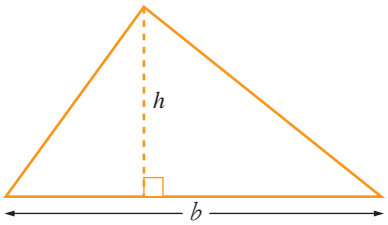
10 If $b = 3t - 1$, find b when:

a $t = 5$ **b** $t = 12$ **c** $t = -9$

See Example 13

11 The formula for the cost of a party is $C = 45n + 500$, where C is the cost in dollars and n is the number of guests.
Use this formula to calculate the cost of a party for 72 guests.

12 The formula for the area of a triangle is $A = \frac{1}{2}bh$, where A is the area of the triangle, b is the length of the base and h is the height.
Find the area of a triangle with base length 9 m and height 6 m.



13 The number of hours of sleep recommended for children has the formula $H = 17 - \frac{A}{2}$, where H is the number of hours and A is the age of the child.
Find the number of hours of sleep recommended for a 13-year-old.

Worked solutions
Exercise 5-05
MAT07NAWS10032

14 The formula for converting Celsius temperatures to Fahrenheit temperatures is $F = \frac{9}{5}C + 32$, where F is the temperature in Fahrehneit and C is the temperature in Celsius.
Use the formula to convert 30°C to Fahrenheit.

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15 A locksmith charges according to this formula $C = 42h + 65$, where C is the charge in dollars and h is the number of hours worked.
How much does the locksmith charge for working for 2 hours?

Technology Substitution

1 Create the spreadsheet below for evaluating five algebraic expressions involving two variables, a and b, whose values are entered in columns A and B.

	A	B	C	D	E	F	G
1	**a**	**b**	**ab**	**a + b**	**2a + b**	**5(a + b)**	**3b ÷ 2a**
2	2	1	=A2*B2				
3	10	0					
4	6	8					
5	-3	5					
6	0.4	1.7					
7	86	45					

2 Column C is for evaluating ab for the different values of a and b shown in rows 2 to 7. In cell C2, enter the formula for calculating ab using the values in A2 and B2.

3 **Fill Down** to copy this formula down to C7.

4 Enter appropriate formulas for the other four expressions in cells D2 to G2. Use brackets where necessary.

5 For each column, D to G, **Fill Down** to row 7.

6 Create the new spreadsheet below that shows a value for each variable, for example, $m = 5$, $n = 2$, $p = -4$.

	A	B	C	D	E	F	G
1	**m**	**n**	**p**	**t**	**u**	**A**	**B**
2	5	2	-4	12	0	50	31
3							

7 In each cell given, enter an appropriate formula to evaluate the algebraic expression shown.

a In cell A4: $m + p$
b In cell B4: npt
c In cell C4: $n + 2p$
d In cell D4: $B - A$
e In cell A5: $(p - n) \div t$
f In cell B5: $np + tu$
g In cell C5: $\frac{mt}{p}$
h In cell D5: $u - At$
i In cell A6: $A^2 - B^2$
j In cell B6: $6p + t$
k In cell C6: $\frac{m(t - n)}{A}$
l In cell D6: $3n^2 - p$

Investigation: Equivalent expressions

Worksheet
Why aren't they the same?
MAT07NAWK10037

1 Determine whether each equation below is true or false by substituting different numbers for the variable and checking whether the left expression equals the right expression for each number.

a $6 + 4m + 4 = 10 + 4m$	**b** $5x + 3x = 8x^2$	**c** $4k - k = 4$
d $7g + 12 = 19g$	**e** $3 + 7n + 2n = 3 + 9n$	**f** $3p \times 7 = 21p$
g $21m \div 3 = 7m$	**h** $2y \times 4y = 24y$	**i** $2r \times 2 \times 2r = 8r^2$

2 For each of the false statements in question **1**, work out what the correct statement should be.

5-06 Equations

Worksheet
Guess-and-check
MAT07NAWK10038

An **equation** is a statement involving a variable (such as x), numbers and an equals ($=$) sign, for example, $\mathbf{4}x + \mathbf{5} = \mathbf{57}$. When we find the correct value that makes the equation true, we **solve** the equation. The value is called the **solution** to the equation.
There are different ways of solving equations. The simplest method is the 'guess, check and improve' method.

The 'guess, check and improve' method

The 'guess, check and improve' method involves:

- Guessing a number for the solution
- Checking the guess by substituting the number into the equation
- Improving on the guess by testing better numbers until the correct solution is found

Example 14

Use 'guess, check and improve' to solve the equation $4x + 5 = 57$ for x.

Solution

Guess	Check	Comment
$x = 3$	$4 \times 3 + 5 = 17$	Smaller than 57. Try a bigger number.
$x = 10$	$4 \times 10 + 5 = 45$	Still smaller than 57. Try a bigger number.
$x = 20$	$4 \times 20 + 5 = 85$	Bigger than 57. Try a number between 10 and 20.
$x = 15$	$4 \times 15 + 5 = 65$	Still bigger than 57. Try a number between 10 and 15.
$x = 13$	$4 \times 13 + 5 = 57$	Correct.

The solution is $\boldsymbol{x = 13}$.

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Exercise 5-06 Equations

1 Solve each equation.

See Example 14

a $x + 3 = 6$	**b** $x - 3 = 6$	**c** $a + 12 = 17$	**d** $m - 5 = 15$
e $b + 1 = 11$	**f** $5c = 20$	**g** $4k = 24$	**h** $d - 3 = 10$
i $10 + m = 25$	**j** $14 - x = 9$	**k** $\frac{y}{3} = 5$	**l** $\frac{n}{5} = 5$

2 What is the solution to $2x - 6 = 24$? Select the correct answer **A**, **B**, **C**, or **D**.

A $x = 12$ **B** $x = 15$ **C** $x = 14$ **D** $x = 9$

3 Solve each equation.

a $2x + 4 = 14$	**b** $3p - 5 = 16$	**c** $4k + 6 = 26$	**d** $5x - 9 = 11$
e $7x + 6 = 27$	**f** $8x - 30 = 34$	**g** $\frac{m}{4} - 3 = 4$	**h** $\frac{a}{2} + 12 = 18$
i $10 - 3d = 4$	**j** $12 - 5n = 7$	**k** $\frac{r - 9}{2} = 3$	**l** $\frac{k + 1}{3} = 4$
m $2p + 3 = -1$	**n** $\frac{r}{5} + 4 = 6$	**o** $\frac{2y}{3} = 10$	**p** $2 \times (x + 1) = 6$

Worked solutions
Exercise 5-06
MAT07NAWS10033

5-07 One-step equations

The 'guess, check and improve' method is simple but slow. There are two **algebraic** methods for solving equations that are faster and more efficient:

- **balancing:** performing the same operation on both sides of the equation
- **backtracking:** undoing each operation by performing the inverse (opposite) operation

Puzzle sheet
Equations dominoes
MAT07NAPS10020

TLF learning object
Algebra balance scales: negatives (L3510)

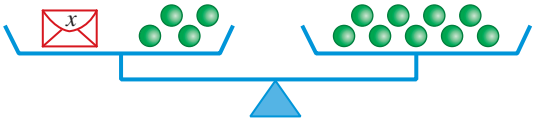
Example 15

Solve the equation $x + 4 = 9$.

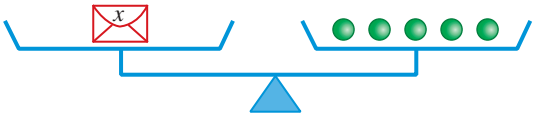
Solution

Method 1: The balancing method

Let x represent an unknown number in an envelope, and ● represent 1. We can represent the equation $x + 4 = 9$ on balance scales like this.



The two sides of the balance are equal. To find the value of x we can remove 4 balls from both sides.



The solution is $x = 5$.

The equation can also be solved algebraically without diagrams.

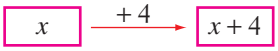
$x + 4 = 9$

$x + 4 - 4 = 9 - 4$ Subtracting 4 from both sides

$\boldsymbol{x = 5}$ Check: $5 + 4 = 9$.

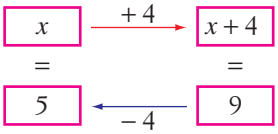
Method 2: The backtracking method

First use a flowchart to show how we get from x to $x + 4$.



In this equation, $x + 4 = 9$. To backtrack (get back) to x, we need to undo the operation 'add 4,' which is 'subtract 4'.

Use a reverse flowchart to undo what has been done to x.



The solution is $\boldsymbol{x = 5}$.

The equation can also be solved algebraically without flowcharts.

$x + 4 = 9$

$x = 9 - 4$ Undo '+ 4' by subtracting 4

$\boldsymbol{x = 5}$ Check: $5 + 4 = 9$.

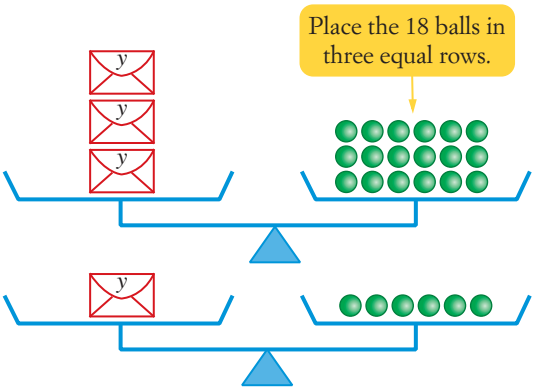
Example 16

Solve $3y = 18$.

Solution

Method 1: The balancing method

Represent the equation $3y = 18$ using balance scales.



To find the value of y, divide both sides by 3.

The solution is $\boldsymbol{y = 6}$.

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Solving this algebraically, we have:

$3y = 18$

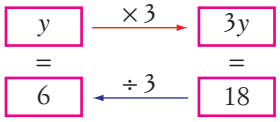
$\frac{3y}{3} = \frac{18}{3}$ Dividing both sides by 3

$\boldsymbol{y = 6}$ Check: $3 \times 6 = 18$

Method 2: The backtracking method

First use a flowchart to show how we get from y to $3y$.

In this equation, $3y = 18$. To backtrack to y, we need to undo the operation '$\times$ 3', which is '$\div$ 3', using a reverse flowchart:



The solution is $y = 6$.

Solving the problem algebraically, we have:

$3y = 18$

$y = \frac{18}{3}$ Undo '$\times$ 3' in $3y$ by dividing by 3

$\boldsymbol{y = 6}$ Check: $3 \times 6 = 18$

Summary

To solve an equation, aim to have the variable (such as x) on one side of the equation and a number on the other side, in the form:

$x =$ _____

Check your solution by substituting it back into the equation.

Example 17

Solve $u - 5 = 7$.

Solution

Method 1: The balancing method

$u - 5 = 7$

$u - 5 + 5 = 7 + 5$ Add 5 to both sides

$\boldsymbol{u = 12}$ Check: $12 - 5 = 7$

Method 2: The backtracking method

$u - 5 = 7$

$u = 7 + 5$ Undo '$-$ 5' by adding 5

$\boldsymbol{u = 12}$ Check: $12 - 5 = 7$

Example 18

Solve $\frac{k}{7} = 2$.

Solution

Method 1: The balancing method

$$\frac{k}{7} = 2$$

$\frac{k}{7} \times 7 = 2 \times 7$ Multiply both sides by 7

$\boldsymbol{k = 14}$ Check: $\frac{14}{7} = 2$

Method 2: The backtracking method

$\frac{k}{7} = 2$

$k = 2 \times 7$ Undo '$\div 7$' by multiplying by 7

$\boldsymbol{k = 14}$ Check: $\frac{14}{7} = 2$

Summary

Operation	Inverse operation
$+$	$-$
$-$	$+$
$\times$	$\div$
$\div$	$\times$

Exercise 5-07 One-step equations

See Example 15

1 Solve each equation, showing the working.

a $w + 8 = 15$ **b** $x + 2 = 20$ **c** $m + 3 = 19$
d $p + 12 = 30$ **e** $x + 13 = 22$ **f** $k + 11 = 20$
g $5 + m = 2$ **h** $6 + y = 16$ **i** $15 + d = 6$

See Example 16

2 Solve each equation, showing the working.

a $3m = 18$ **b** $5n = 20$ **c** $4k = 44$
d $2c = 32$ **e** $9x = 81$ **f** $3x = -27$
g $7d = 42$ **h** $6h = -30$ **i** $10a = 70$

See Example 17

Worked solutions
Exercise 5-07
MAT07NAWS10034

3 Solve each equation, showing the working.

a $p - 3 = 8$ **b** $m - 11 = 2$ **c** $x - 5 = 12$
d $y - 20 = 40$ **e** $k - 21 = 3$ **f** $n - 3 = 18$
g $7 = d - 11$ **h** $y - 13 = -4$ **i** $-5 = m - 9$

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4 Solve each equation, showing the working.

See Example 18

a $\frac{m}{3} = 4$ **b** $\frac{d}{2} = 5$ **c** $\frac{x}{5} = 8$ **d** $\frac{k}{4} = 12$

e $\frac{x}{10} = 6$ **f** $\frac{a}{2} = -4$ **g** $15 = \frac{k}{2}$ **h** $-11 = \frac{n}{5}$

5 Solve each equation, showing the working.

a $m - 3 = 16$ **b** $3 + p = 11$ **c** $24 = p + 5$

d $n + 7 = -1$ **e** $3x = 30$ **f** $n - 14 = -5$

g $3x = 21$ **h** $4y = -18$ **i** $\frac{h}{7} = 1$

j $-2x = -6$ **k** $6 + k = -4$ **l** $\frac{n}{4} = 0$

Worked solutions
Exercise 5-07
MAT07NAWS10034

5-08 Two-step equations

The following 'two-step' equations require two operations in finding their solutions.

Homework sheet
Equations 1
MAT07NAHS10022

Example 19

Solve the equation $2x + 7 = 9$.

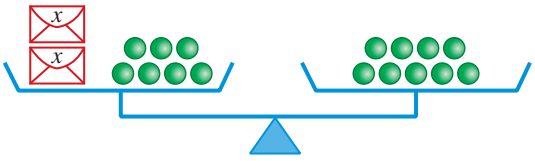
Video tutorial
Two-step equations
MAT07NAVT10012

TLF learning object
Algebra balance scales: negatives (L3510)

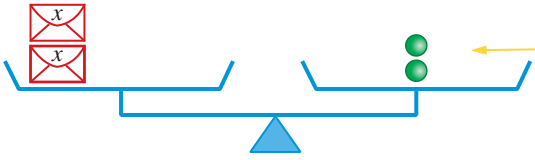
Solution

Method 1: The balancing method

We can represent $2x + 7 = 9$ using balance scales.

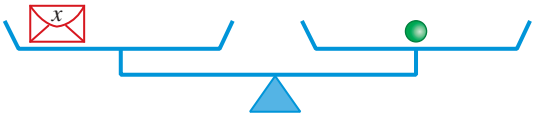


First subtract 7 balls from both sides.



Place the remaining balls into two equal rows.

Then divide both sides by 2.



The solution is $\boldsymbol{x = 1}$.

Using algebra we have: $2x + 7 = 9$

$2x + 7 - 7 = 9 - 7$ Step 1: Subtracting 7 from both sides

$2x = 2$

$\frac{2x}{2} = \frac{2}{2}$ Step 2: Dividing both sides by 2

$\boldsymbol{x = 1}$

Check: $2 \times 1 + 7 = 9$

Worksheet
Backtracking
MAT07NAWK10039

Worksheet
Backtracking
MAT07NAWK00003

Puzzle sheet
Solving equations
MAT07NAPS00006

Video tutorial
Solving equations
MAT07NAVT00006

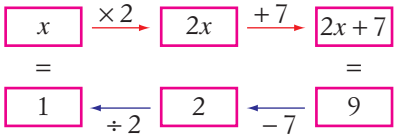
Worksheet
Equations match
MAT07NAWK10040

Worksheet
Equations writing activity
MAT07NAWK10041

Method 2: The backtracking method

Use a flowchart to go from x to $2x + 7$.

$2x + 7 = 9$. To backtrack, we need to undo the operations '$\times$ 2' and '$+$ 7' in **reverse order**, so '$-$ 7' and '$\div$ 2' using a reverse flowchart.



The solution to $2x + 7 = 9$ is $x = 1$.

Using algebra, we have: $2x + 7 = 9$

$2x = 9 - 7$ Step 1: Undo '$+$ 7' by subtracting 7

$2x = 2$

$x = \frac{2}{2}$ Step 2: Undo '$\times$ 2' in $2x$ by dividing by 2

$\boldsymbol{x = 1}$ Check: $2 \times 1 + 7 = 9$

Note that inverse operations are performed in **reverse order**. For example, to undo putting on our socks and shoes, we take off our shoes first, then our socks.

Example 20

Solve $\frac{M}{3} + 2 = 5$.

Solution

Method 1: The balancing method

$\frac{M}{3} + 2 = 5$

$\frac{M}{3} + 2 - 2 = 5 - 2$ Step 1: Subtracting 2 from both sides

$\frac{M}{3} = 3$ Simplify.

$\frac{M}{3} \times 3 = 3 \times 3$ Step 2: Multiplying both sides by 3

$\boldsymbol{M = 9}$ Check: $\frac{9}{3} + 2 = 3 + 2 = 5$

Method 2: The backtracking method

$\frac{M}{3} + 2 = 5$

$\frac{M}{3} = 5 - 2$ Step 1: Undo '$+$ 2' by subtracting 2.

$\frac{M}{3} = 3$

$M = 3 \times 3$ Step 2: Undo '$\div$ 3' by multiplying by 3.

$\boldsymbol{M = 9}$ Check: $\frac{9}{3} + 2 = 3 + 2 = 5$

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Exercise 5-08 Two-step equations

1 Solve each equation showing all steps. Remember to check your answers.

See Example 19

a $2x + 1 = 11$	**b** $3x + 8 = 17$	**c** $4x + 5 = 13$
d $4x + 9 = 25$	**e** $5d + 2 = 47$	**f** $2x + 10 = -6$
g $-3x + 10 = 4$	**h** $-4x + 9 = 5$	**i** $-5x + 8 = -12$
j $-7x + 4 = -17$	**k** $-2x + 7 = 9$	**l** $-4x + 14 = 22$

Worked solutions
Exercise 5-08
MAT07NAWS10035

2 The following is Liam's incorrect solution for $7x + 5 = 13$.

$7x + 5 = 13$
$7x = 13 - 5$ Line 1
$7x = 8$ Line 2
$x = \frac{8}{7}$ Line 3
$x = 1\frac{1}{8}$ Line 4

In which of the following was the error made? Select **A**, **B**, **C** or **D**.

A Line 1 **B** Line 2 **C** Line 3 **D** Line 4

3 Solve each equation showing all steps.

a $2x - 5 = 9$	**b** $3x - 1 = 23$	**c** $5x - 3 = 12$
d $2x - 4 = 18$	**e** $6x - 4 = 8$	**f** $5x - 19 = -9$
g $4x - 7 = 5$	**h** $7x - 3 = 32$	**i** $2x - 4 = -14$
j $5x - 2 = 43$	**k** $3x - 4 = -10$	**l** $8x - 12 = -20$

Worked solutions
Exercise 5-08
MAT07NAWS10035

4 What is the solution for $\frac{n}{4} - 8 = 7$? Select the correct answer **A**, **B**, **C** or **D**.

A $n = 4$ **B** $n = 20$ **C** $n = 36$ **D** $n = 60$

5 Solve each equation.

See Example 20

a $\frac{x}{2} + 5 = 9$	**b** $\frac{m}{3} + 6 = 9$	**c** $\frac{k}{5} + 2 = 6$
d $\frac{a}{4} + 4 = 8$	**e** $\frac{n}{2} + 6 = 14$	**f** $\frac{h}{7} + 2 = -3$
g $\frac{m}{2} - 8 = -5$	**h** $\frac{k}{5} - 7 = -4$	**i** $\frac{x}{3} - 6 = -10$
j $\frac{a}{3} - 1 = 2$	**k** $\frac{h}{5} - 4 = -7$	**l** $\frac{x}{2} - 3 = 4$

Mental skills 5B Maths without calculators

Multiplying by 9, 11 and 12

To multiply a number by 9, multiply by 10 and then subtract the number.
This is because 10 times a number minus the same number equals 9 times the number.

1 Study each example.

a $14 \times 9 = 14 \times 10 - 14 = 140 - 14 = 126$
b $25 \times 9 = 25 \times 10 - 25 = 250 - 25 = 225$
c $18 \times 9 = 18 \times 10 - 18 = 180 - 18 = 162$

2 Now simplify each product.

a 12×9 **b** 27×9 **c** 46×9 **d** 19×9
e 34×9 **f** 63×9 **g** 21×9 **h** 15×9

To multiply a number by 11, multiply by 10 and then add the number.
This is because 10 times a number plus the same number equals 11 times the number.

3 Study each example.

a $26 \times 11 = 26 \times 10 + 26 = 260 + 26 = 286$
b $13 \times 11 = 13 \times 10 + 13 = 130 + 13 = 143$
c $35 \times 11 = 35 \times 10 + 35 = 350 + 35 = 385$

4 Now simplify each product.

a 17×11 **b** 22×11 **c** 38×11 **d** 40×11
e 25×11 **f** 19×11 **g** 54×11 **h** 31×11

To multiply a number by 12, multiply by 10, then add double the number.
This is because 10 times a number plus double the same number equals 12 times the number.

5 Study each example.

a $22 \times 12 = 22 \times 10 + 22 \times 2 = 220 + 44 = 264$
b $16 \times 12 = 16 \times 10 + 16 \times 2 = 160 + 32 = 192$
c $70 \times 12 = 70 \times 10 + 70 \times 2 = 700 + 140 = 840$

6 Now simplify each product.

a 44×12 **b** 15×12 **c** 29×12 **d** 31×12
e 52×12 **f** 18×12 **g** 26×12 **h** 37×12

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Just for the record — Ancient Egyptian equations

The Rhind Papyrus from ancient Egypt is one of the earliest mathematical documents we have, dating back to 1650 BCE. It now belongs to the British Museum, and contains the following equation written in hieroglyphics (Egyptian picture symbols).

The algebraic translation is: $x + \frac{2}{3}x - \frac{1}{3}\left(x + \frac{2}{3}x\right) = 10$

Note that legs walking left mean 'minus'  , legs walking right mean 'add' .

Find out why the Rhind Papyrus has that name.

5-09 Equation problems

Homework sheet
Equations 2
MAT07NAHS10023

Many mathematical problems are stated in words. We can solve these problems by translating the problems into algebraic symbols and equations.

Example 21

When a number is doubled and then 5 is added, the answer is 69. Find the number.

Solution

Let the number be x.

Translating the problem into an equation:

$x \times 2 + 5 = 69$

$2x + 5 = 69$

Solve the equation.

$2x + 5 - 5 = 69 - 5$ — Subtracting 5 from both sides

$2x = 64$

$\frac{2x}{2} = \frac{64}{2}$ — Dividing both sides by 2

$x = 32$

Check: $2 \times 32 + 5 = 69$

The number is 32.

Example 22

Tom is 8 years older than Susi. If the sum of their ages is 22, find their ages.

Solution

Let Susi's age be n.

So Tom's age is $n + 8$.

	Tom is 8 years older.
$n + n + 8 = 22$	Susi's age + Tom's age = 22
$2n + 8 = 22$	Simplifying $n + n$ to $2n$
$2n + 8 - 8 = 22 - 8$	Subtracting 8 from both sides
$2n = 14$	
$\frac{2n}{2} = \frac{14}{2}$	Dividing both sides by 2
$n = 7$	Check: $2 \times 7 + 8 = 22$
So Susi is 7 and Tom is $7 + 8 = 15$.	Check: $7 + 15 = 22$

Summary

For word problems involving equations:

- Choose a pronumeral
- Translate the problem into an equation
- Solve the equation
- Write a sentence that answers the problem

Exercise 5-09 Equation problems

See Example 21

1 Solve each problem by writing an equation and then solving it. You may use diagrams to help you think about the information.

a Five tickets for a film cost $55. How much does each ticket cost? (Let t represent the price of a ticket.)

b Ten oranges cost $4.80. How much does each orange cost? (Let x represent the cost of one orange.)

c A number is doubled and the result is 110. What is the number? (Let n represent the unknown number.)

d A number has 4 subtracted from it and the result is 6. Find the number. (Let y represent the unknown number.)

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2 For each word problem, select the correct equation **A**, **B**, **C** or **D**. Then solve the equation to solve the problem.

a Jarrad has collected 1794 beetles. This is 6 times as many beetles as Lisa has in her collection. How many beetles does Lisa have?

A $1794 - N = 6$ **B** $6N = 1794$ **C** $N = 6 \times 1794$ **D** $N + 6 = 1794$

b Kurt mixed 590 mL of white paint with some blue paint. He mixed 1.73 L of paint altogether. How much blue paint did he use?

A $N + 590 = 1730$ **B** $N - 590 = 1730$ **C** $\frac{N}{590} = 1730$ **D** $590N = 1730$

c Fourteen packets of chocolate biscuits are packed in a box. The supermarket sold 546 packets of biscuits. How many boxes were sold?

A $14N = 546$ **B** $546 - N = 14$ **C** $N + 14 = 546$ **D** $\frac{N}{14} = 546$

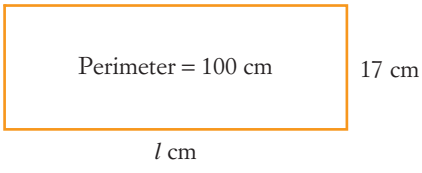
d When a certain number is subtracted from 100, the result is 47. What is the number?

A $N - 100 = 47$ **B** $100 + 47 = N$ **C** $N - 47 = 100$ **D** $100 - N = 47$

3 Translate each of the following problems into an equation, then solve the equation to solve the problem. *See* **Example 22**

a Mr Abdul says, 'If you multiply my age by 4 and add 12, the answer is 240.' How old is Mr Abdul? (Let a stand for his age.)

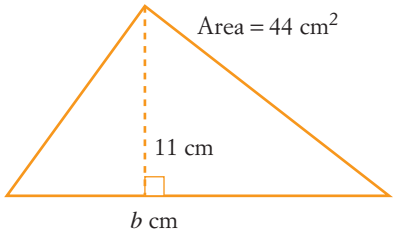
b The perimeter of a rectangle is 100 cm and its width is 17 cm. What is the length? (Let l cm represent the length of the rectangle.)



c I think of a number. If I multiply it by 6 and subtract 13, the answer is 95. What is the number? (Let y represent the number.)

Worked solutions
Exercise 5-09
MAT07NAWS10036

d The area of a triangle is calculated by multiplying the base by the height and dividing by 2. If a triangle has an area of 44 cm^2 and its height is 11 cm, what is the length of its base? (Let b cm represent the length of the base.)



e The Student Representative Council is holding a school disco to raise money. Each ticket bought by students raises $7 but the cost of running the disco is $184. How many tickets must be sold to make a profit of $2000? (Let n stand for the number of tickets sold.)

f Grace is a salesperson who earns $200 per week plus one-fifth of the value of her sales for that week. If she is paid $750 for one week, what is the value of her sales for that week? (Let x stand for the value of her sales.)

4 A repairman charges for fixing washing machines using the formula $C = 32h + 45$, where C is the charge in dollars and h is the number of hours the job takes.
Find the number of hours worked if the charge is $205.

5 The weekly profit, in dollars, made by a DVD store is given by the formula $P = 5D - 900$, where D represents the number of DVDs hired. Find the number of DVDs hired if the profit is $1055.

6 The volume of a rectangular prism is given by the formula $V = lbh$. What is the value of l when $V = 340$, $b = 4$ and $h = 5$? Select the correct answer **A**, **B**, **C** or **D**.

A 17 **B** 68 **C** 85 **D** 425

Investigation: School concert

Year 7 wants to hold a school concert to help subsidise the cost of their camp. It wants to raise $1500 for this.
The concert will cost $100 for each night it is on. The hall will hold 250 people. If a band is hired for the concert this would cost an extra $550 per night.

a If Year 7 decide to stage one concert only without a band, calculate how much each ticket should be to cover the costs and raise the required amount. Use an equation to help you.

b If they decide to hire the band, write another equation to find the new ticket price.

c If the Student Representative Council decides to have three concerts with the band, calculate what the ticket price should be. Consider how many people you think would attend each night. Write another equation and solve it to justify your answer.

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Power plus

1 Simplify each expression.

a $2a + b + 3a - 4b$ **b** $m - 3n - 5m - 7n$

c $3k - 2j - 5 - 4k + 6j + 9$ **d** $a^2 + 2b^2 + 3b^2 - a^2$

e $4x^2 + 6x - x^2 - 12x$ **f** $xy + yz + xz + 4xy + 3xz + yz$

2 Evaluate each expression if $p = 4$, $q = -6$, $r = 3$ and $t = -1$.

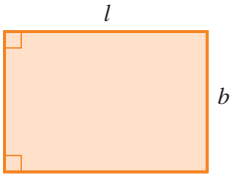
a $p + q$ **b** pq **c** $r^2 + t^2$ **d** $q^2 - 6p + rt$

e $\frac{pq}{rt}$ **f** $6 - 2r + p$ **g** $2p + 3q + 4r - 5t$ **h** $\frac{p+4}{2}$

3 For each shape, write an algebraic expression for:

i its perimeter **ii** its area

a



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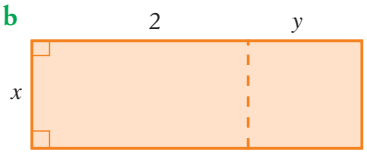

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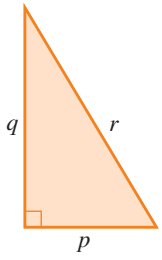

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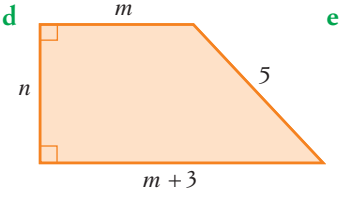

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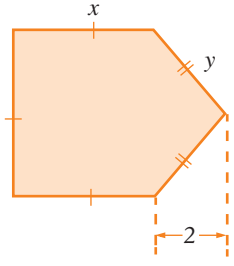
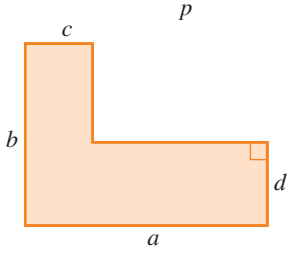


f



4 If x is any number, then simplify each expression.

a $x + 0$ **b** $x \times x$ **c** $0 \div x$ **d** $x \div x$ **e** $x - (-x)$

f $0 \times x$ **g** $0 - x$ **h** $(-x)^2$ **i** $x \times \frac{1}{x}$ **j** $x - 2x$

5 Solve each equation.

a $6 - 3x = 9$ **b** $9 - 8k = 33$ **c** $7 - 2n = 11$

d $2(r - 4) = 10$ **e** $3(d + 6) = 18$ **f** $4(z + 1) = -6$

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Chapter 5 review

Language of maths

Puzzle sheet
Algebra find-a-word
MAT07NAPS10021

abbreviation	commutative law	evaluate	pronumeral
algebra	consecutive	formula	solve
associative law	distributive law	guess, check and improve	solution
backtracking	equation		substitution
balancing	expanded form	inverse	two-step
check	expression	one-step	variable

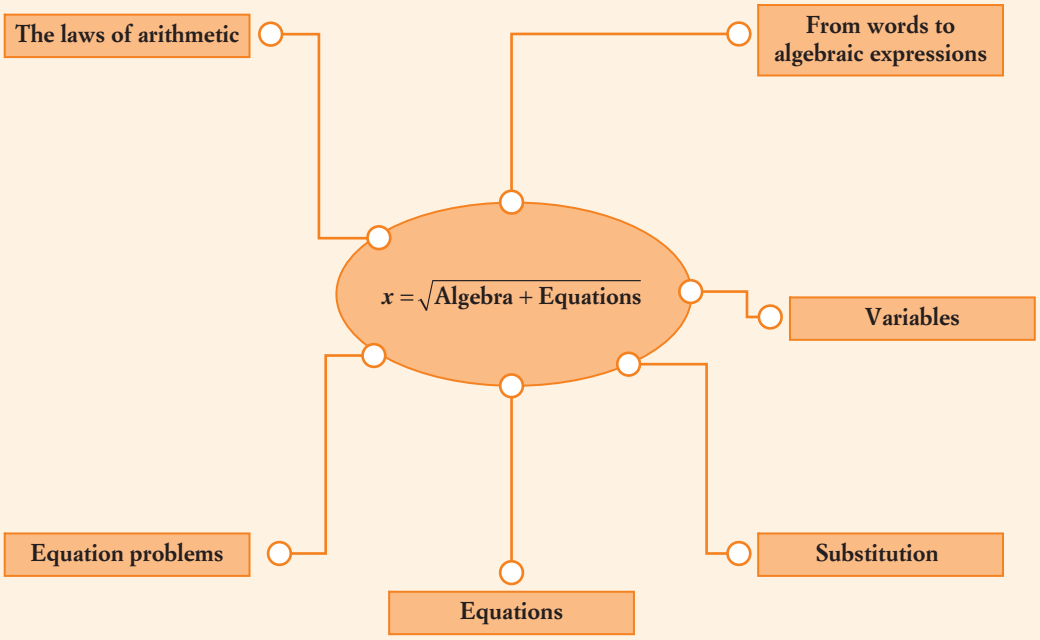
1 What is another name for **pronumeral**?
2 What is the difference between an **expression** and an **equation**?
3 Which law involves multiplying by a number by splitting that number to make the calculation easier?
4 What are **consecutive numbers**?
5 Which method for solving equations is associated with 'undoing' operations?
6 What name is given to the value that makes an equation true?

Topic overview

Worksheet
Mind map: Algebra and equations
MAT07NAWK10042

- Which parts of this topic did you find easy? Was there anything you already knew?
- Are there parts of this chapter that you still do not understand? Talk to your teacher.
- Give two examples of jobs where algebra or equations might be needed.

Print (or copy) and complete this mind map of the topic, adding detail to its branches and using pictures, symbols and colour where needed. Ask your teacher to check your work.



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Chapter 5 revision

1 Use the commutative and associative laws to evaluate each expression. *See* Exercise 5-01

a $7 + 18 + 3$ **b** $91 + 18 + 9$ **c** $2 \times 17 \times 5$

d $25 \times 31 \times 4$ **e** 13×30 **f** 7×400

2 Use the distributive law to evaluate each expression. *See* Exercise 5-02

a 22×9 **b** 17×8 **c** 27×11 **d** 13×12

3 Examine the number pattern below, then write the general rule using a variable. *See* Exercise 5-03

$3 - (-3) = 2 \times 3$ $11 - (-11) = 2 \times 11$

$5 - (-5) = 2 \times 5$ $2 - (-2) = 2 \times 2$

4 Simplify each expression. *See* Exercise 5-03

a $x + x + x + x$ **b** $u \div 5$ **c** $2y - y$

c $5 \times a \times 2$ **d** $8 \times c \times b \times 3 \times a$ **e** $2 \times m \times w \times 10 \times w$

5 Write each of these in expanded form. *See* Exercise 5-03

a $4mnk$ **b** $\frac{6a}{b}$ **c** $9c^2$

6 Write an algebraic expression for each statement. Use N to represent any number. *See* Exercise 5-04

a 3 times the number **b** the difference between the number and 5

c the next consecutive number **d** one-third of the number

7 Write an expression for: *See* Exercise 5-04

a the sum of M and 3 **b** 5 more than B **c** $2H$ decreased by k

8 Find the value of these expressions if $a = 2$, $b = 5$ and $c = 6$. *See* Exercise 5-05

a $a + b$ **b** $3a - c$ **c** $a + 2b - c$ **d** abc

9 If $C = 30h + 19$, find C when *See* Exercise 5-05

a $h = 3$ **b** $h = -1$ **c** $h = 5$ **d** $h = 2.5$

10 Solve each equation using the 'guess, check and improve' method. *See* Exercise 5-06

a $4m - 9 = 7$ **b** $\frac{k+2}{7} = 2$ **c** $\frac{d}{3} - 4 = 2$

11 Solve each equation algebraically, showing all steps. *See* Exercise 5-07

a $w + 17 = 52$ **b** $e - 12 = 0$ **c** $5p = 15$ **d** $\frac{x}{2} = 6$

12 Solve each equation algebraically, showing all steps. *See* Exercise 5-08

a $3t + 4 = 10$ **b** $2n + 6 = 12$ **c** $4y - 7 = 5$

d $\frac{m}{4} - 8 = 9$ **e** $\frac{x}{3} + 5 = 8$ **f** $\frac{m-4}{2} = 3$

13 **a** When a certain number is multiplied by 17, the product is 1003. Which equation can be used to solve this problem? (Let N represent the number.) Select **A**, **B**, **C** or **D**. *See* Exercise 5-09

A $N + 17 = 1003$ **B** $1003 - N = 17$ **C** $17N = 1003$ **D** $1003 + N = 17$

b Solve the equation to solve the problem.

14 Caitlyn organises a charity raffle at her school. The only cost is \$180 for prizes. *See* Exercise 5-09

a If she sets the price of raffle tickets at \$5, write an equation to find out how many tickets she must sell to just cover costs. (Let N represent the number of tickets.)

b To make a profit of \$2100, how many tickets must she sell?